

MIT-CTP-2624

hep-th/9704196

April 1997

Supersymmetric Dilatations in the Presence of Dilaton

HoSeong La*

*Center for Theoretical Physics**Laboratory for Nuclear Science**Massachusetts Institute of Technology**77 Massachusetts Avenue**Cambridge, MA 02139-4307, USA*

The supersymmetric generalization of dilatations in the presence of the dilaton is defined. This is done by defining the supersymmetric dilaton geometry which is motivated by the supersymmetric volume preserving diffeomorphisms. The resulting model is classical superconformal field theory with an additional dilaton-axion supermultiplet coupled to the supersymmetric gauge theory, where the dilaton-axion couplings are nonrenormalizable. The possibility of spontaneous scale symmetry breaking is investigated in this context. There are three different types of vacua with broken scale symmetry depending on the details of the dilaton sector: unbroken supersymmetry, spontaneously broken supersymmetry and softly broken supersymmetry. If the scale symmetry is broken in the bosonic vacuum, then the Poincaré supersymmetry must be broken at the same time. If the scale symmetry is broken in the fermionic vacuum but the bosonic vacuum remains invariant, then the Poincaré supersymmetry can be preserved as long as the R-symmetry breaking is specifically related to the scale symmetry breaking.

PACS: 11.30 11.30.Ly 11.30.Pb 11.30.Qc

*e-mail address: hsla@mitlms.mit.edu

This work is supported in part by funds provided by the U.S. Department of Energy (D.O.E.) under cooperative research agreement #DF-FC02-94ER40818.

1. Introduction

The dilaton is a hypothetical particle which is a Goldstone boson associated with spontaneous breaking of scale symmetry as an analog to the pion for chiral symmetry breaking[1]. Scale symmetry is one of the most fascinating concepts in physics despite the fact that nature does not seem to respect it at all at least in the macroscopic scale. Microscopically, since the MIT-SLAC deep inelastic scattering experiments[2] the role of dilatations in physics has attracted fair amount of attention. Knowing the fact that we live in the world of a given scale, the classical scale symmetry[3][4][5][6] based on the dilatations of local coordinates in a Lorentz frame is destined to be broken at that given scale. Scale symmetry breaking in principle can be either explicit or spontaneous. It turns out that in a simple model with a scalar field the classical scale symmetry is anomalous at the quantum level, hence it is explicitly broken[5]. In more realistic cases like massless QED or gauge theories, scale symmetry is also broken by the trace anomaly[7]. This does not allow any room for the dilaton to be introduced. Nevertheless, there have been attempts to introduce spontaneous breaking of scale symmetry[1].

If we look into the matter more carefully, the existence of anomalies does not necessarily rule out any possibility of spontaneous symmetry breaking. If spontaneous symmetry breaking occurs in a sector different from the anomalous sector, it is possible. In fact, a good example is the case of the axion. Despite the axial anomaly, we can obtain spontaneous breaking of the Peccei-Quinn symmetry and that the axion can be generated in QCD[8]. This seems to be the case for the dilaton too. Furthermore, the dilaton does not transform like a quasiprimary field under dilatations, it does not follow the standard structure of the renormalization group either. So, in the presence of the dilaton the renormalization group argument does not rule out the possibility of spontaneous breaking of scale symmetry.

The dilaton appears more commonly in the context of gravity, although it often lacks an unambiguous way of defining the dilaton. Not all scalar fields appearing in gravitational models are dilatons, but the only one related to the spontaneous breaking of scale symmetry is the dilaton. For example, in Kaluza-Klein approaches a compactification of $(n + 1)$ down to n dimensional spacetime introduces a scalar field which behaves like a dilaton in n dimensions. But this is not a definition of the dilaton but just a coincidence because $(n + k)$ -to- n compactifications can lead to a set of scalar fields satisfying nonlinear σ -models, none of which has the property of the dilaton[9]. The reason we sometimes consider this scalar field as a dilaton is the coincidence observed in the low energy effective supergravity models of string theory.

A low energy effective ten dimensional supergravity action of string theory can be derived from an eleven dimensional supergravity model action via compactification on a circle and the Kaluza-Klein scalar field appears as the dilaton in the Einstein frame derived from the string theoretical dilaton[11]. In string theory the dilaton is more precisely and rather unambiguously defined as follows: The dilaton in string theory is the spacetime scalar field coupled to the world-sheet curvature scalar in the world-sheet nonlinear σ -model action[12]. This is the only massless scalar field and the theory has classical scale symmetry, hence it has to be the gravitational dilaton. This leads to the Kaluza-Klein scalar field in the Einstein frame, in which the metric is a combination of the stringy dilaton and the spacetime metric in the string frame, that is, the spacetime metric defined in the worldsheet nonlinear σ -model action. Thus the dilaton that appears in the low energy effective supergravity action is not really independent from the trace of the graviton, which often leads to dilaton fluctuations involving the trace of the graviton[13][14].

To further understand the role of scale symmetry in nature, we need to address the origin of the low energy scale symmetry. It is commonly believed that the scale symmetry in Minkowski space is an analog to the (rigid) Weyl symmetry in curved spacetime¹. This however is in some sense unsatisfactory because of lack of any direct relationship. This is all right if they are two independent symmetries. However, it is most unlikely that they are unrelated because classical symmetries should not depend on the strength of the gravity, yet the rigid Weyl ceases to make sense when the gravity is turned off. Furthermore, low energy scale transformations involve changes of local coordinates, but Weyl transformations do not. We need a more direct connection between properties in curved spacetime and those in a local Lorentz frame particularly for any spacetime symmetries. This becomes an important issue if gravitational effects get stronger. Particularly, in string-motivated supersymmetric models the dynamics of the dilaton is crucial to understand the structure of the coupling constants and supersymmetry in the low energy[11][15][16][17].

As a matter of fact, the rigid Weyl transformations can be reproduced by diffeomorphisms. Hence, in my previous paper[18] I started only with Diff (diffeomorphism) symmetry without referring to the Weyl symmetry, then derived the scale symmetry in Minkowski spacetime. Note that in any dimensional spacetime scale invariance does not necessarily imply Weyl invariance, although it often implies conformal invariance. This signals that it would be better to understand scale symmetry as part of Diff, and that it could provide a natural explanation

¹See [19], for example.

of the relation between scale symmetry and conformal invariance. Diff decomposes into SDiff (volume-preserving diffeomorphisms) and CDiff (conformal diffeomorphisms)². Since SDiff preserves a volume element, dilatations are not part of SDiff. This is the crucial structure used in ref.[18] and we shall generalize it to the supersymmetric case in this paper.

Dilatations are defined in terms of local coordinates by

$$x \rightarrow e^\alpha x, \quad (1.1)$$

$$\Phi_{[d]}(x) \rightarrow e^{d\alpha} \Phi_{[d]}(e^\alpha x), \quad (1.2)$$

where d is the scale dimension (or the conformal weight). Eq.(1.1) suggests dilatations should be expressed as diffeomorphisms, although eq.(1.2) is not a result of a diffeomorphism. We however find that $\Phi_{[d]}$ can be expressed as a dilaton-dressed field and that eq.(1.2) indeed becomes a result of a diffeomorphism. This can be done only in the presence of the dilaton so that in this context the scale symmetry naturally incorporates the dilaton. As an important result, scale invariance automatically guarantees conformal invariance because both are just part of Diff invariance.

The idea of defining dilatations in the presence of the dilaton without referring to the Weyl transformation involves the diffeomorphism (Diff) symmetry of the dilaton geometry defined by the metric $g_{\mu\nu} = e^{2\kappa\phi} \eta_{\mu\nu}$, where $\phi \equiv \frac{1}{2\kappa n} \ln |g|$ in n -dimensional spacetime. One of the advantages of this new proposal is that dilatations in Minkowski spacetime are naturally related to the symmetry in curved spacetime, while the usual way of thinking that the low energy dilatations are related to the Weyl geometry of curved spacetime actually lacks any direct relationship. The dilaton geometry realizes the conformal symmetry in the presence of the dilaton as much as the conformal geometry does for the conformal symmetry without the dilaton. I expect that this could be the right framework to investigate the dilaton physics in the low energy limit of unified theories that incorporates the dilaton, e.g. string theory.

This paper is organized as follows: In section two, the dilaton geometry is defined. In section three, the $N = 1$ superconformal vector fields are explicitly derived, which are needed to generalize the dilaton geometry to the supersymmetric case in section four. In section five, a superconformally invariant effective lagrangian is derived for the supersymmetric dilaton geometry. In section six the possibility of spontaneous breaking of the scale symmetry in this context is examined. Finally, some perspectives are discussed in the conclusions and one appendix that explains the basics of volume-preserving Diff and its supersymmetric generalization are given.

²For details of SDiff, see [20].

2. Dilaton Geometry and Conformal Symmetry

In curved spacetime, infinitesimal Weyl transformations are given by

$$\delta g_{\mu\nu} = 2\epsilon\rho(x)g_{\mu\nu}, \quad (2.1)$$

for arbitrary function $\rho(x)$. $\rho(x)$ is constant for a global (or rigid) Weyl transformation. Usually in literatures this global Weyl transformation is regarded as the analog to a scale transformation in Minkowski space, hence relating scale symmetry to Weyl symmetry. Weyl transformations are independent from coordinate changes contrary to scale transformations. Thus, by simply taking the flat limit Weyl symmetry does not naturally lead to the scale symmetry.

Spacetime transformations involving local coordinates are diffeomorphisms, commonly known as general coordinate transformations. Under Diff, fields transform according to

$$\left(T_{\mu_1\cdots\mu_p} + \delta T_{\mu_1\cdots\mu_p}\right) dx^{\mu_1} \cdots dx^{\mu_p} = T_{\mu_1\cdots\mu_p}(x + \delta x) d(x + \delta x)^{\mu_1} \cdots d(x + \delta x)^{\mu_p}. \quad (2.2)$$

Then $\delta T_{\mu_1\mu_2\cdots\mu_p}$ is nothing but the Lie derivative along δx . Diff acts on the metric, for $v \equiv \delta x$ in n dimensions, as

$$\delta g_{\mu\nu} = \nabla_\mu v_\nu + \nabla_\nu v_\mu. \quad (2.3)$$

In particular, if v^μ is a conformal Killing vector, then $\delta g_{\mu\nu}$ takes the form of eq.(2.1) with $\rho(x) = \frac{1}{n}\nabla_\mu v^\mu$ and these transformations are conformal diffeomorphisms. Contrary to Weyl transformations, where $\rho(x)$ is arbitrary, such v^μ must exist in this case. If so, one can say CDiff is a special case of Weyl. If v^μ exists for constant ρ , the scale transformation based on the Weyl is also nothing but Diff.

In the presence of the dilaton we can take advantage of the above to the most. The dilaton geometry is defined by a metric in the form of

$$g_{\mu\nu} = e^{2\kappa\phi}\eta_{\mu\nu}, \quad (2.4)$$

where $e^{n\kappa\phi} = \sqrt{g}$ in terms of $g \equiv |\det g_{\mu\nu}|$ and κ^{-1} is the dilaton scale. The effect of introducing an explicit scale parameter, κ , is to let the dilaton have mass dimension $(n-2)/2$, where n is the dimension of spacetime. Note that κ is not really a free parameter because we can always rescale it by rescaling ϕ . As far as gravity is concerned, the natural choice of this scale is the Planck scale. But, here, instead of doing that, we will fix it later at any phenomenologically proper scale so that we can study the dilaton in an energy scale much lower than the quantum gravity

scale. Since κ always appears in combination with ϕ , fixing κ actually requires a nontrivial dilaton vacuum expectation value.

As emphasized in ref.[20], if ϕ does not transform like a scalar under Diff, but the transformation property under Diff is dictated by that of the metric, then eq.(2.3) leads to

$$\delta e^{\kappa\phi} = \frac{1}{n} e^{\kappa\phi} D_\mu v^\mu, \quad (2.5)$$

where

$$D_\mu \equiv \partial_\mu + n\kappa \partial_\mu \phi. \quad (2.6)$$

In terms of g , $D_\mu \equiv \partial_\mu + \partial_\mu \ln \sqrt{g}$ and $\partial_\mu (\sqrt{g} v^\mu) = \sqrt{g} D_\mu v^\mu$. D_μ is the same as the covariant derivative ∇_μ only when it acts on a covariant vector, but, in general, they are different. Eq.(2.5) shows that eq.(2.4) is not to be considered as a conformal gauge fixing condition globally, but is a local expression of a metric in terms of a non-global function ϕ . For example, a density is not global because it depends on a choice of local coordinates. If eq.(2.4) were the conformal gauge fixing condition, it would lead to $\delta e^{2\kappa\phi} = v^\mu \partial_\mu e^{2\kappa\phi}$. Under SDiff, ϕ behaves like a constant to make pure ϕ Lagrangians manifestly SDiff-invariant.

It is important to notice that the consistency condition between eq.(2.5) and eq.(2.4) is

$$\frac{2}{n} \eta_{\mu\nu} \partial_\alpha v^\alpha = \eta_{\mu\alpha} \partial_\nu v^\alpha + \eta_{\alpha\nu} \partial_\mu v^\alpha. \quad (2.7)$$

Hence, diffeomorphisms of eq.(2.4) appear as conformal transformations of the flat spacetime. This shows that the dilaton geometry describes the conformal geometry of the flat spacetime in the presence of the dilaton.

Under dilatations $v^\mu = \alpha x^\mu$, eq.(1.1), the dilaton transforms for infinitesimal α , as

$$\delta\phi = \alpha \left(\frac{1}{\kappa} + x^\mu \partial_\mu \phi \right), \quad (2.8)$$

which is nothing but the dilatation property given in ref.[1]. Note that the dilaton is not a (quasi)primary field, since it does not transform homogeneously. This distinguishes the dilaton from other fields. This also shows that the dilatations of the dilaton are results of diffeomorphisms. Sometimes, it is useful to introduce a field redefinition

$$\chi \equiv e^{\kappa\phi}. \quad (2.9)$$

Under Diff, χ transforms in a not-so-inspiring way, but, under dilatations

$$\delta\chi = \alpha (1 + x^\mu \partial_\mu) \chi. \quad (2.10)$$

Thus, although χ is not a scalar, it transforms like a scale-dimension-one field. χ is mass-dimensionless.

To produce eq.(1.2) let us introduce a dilaton-dressed field $\Phi_{[d]}$ as

$$\Phi_{[d]} \equiv e^{d\kappa\phi}\Phi, \quad (2.11)$$

where Φ transforms like a scalar under Diff. This dilaton dressing does not change the mass dimension of the field. Then under dilatations

$$\delta\Phi_{[d]} = \alpha(d + x^\mu\partial_\mu)\Phi_{[d]} \quad (2.12)$$

so that $\Phi_{[d]}$ is a (quasi)primary field[21]. Such dressing is not needed for vector fields in four dimensions because under dilatations

$$\delta A_\mu = \alpha(1 + x^\lambda\partial_\lambda)A_\mu. \quad (2.13)$$

This in particular leads to the YM term that does not couple directly to the dilaton in four dimensions, hence different from the Kaluza-Klein case. In other than four dimensions vector fields still need dilaton dressing, leading to direct YM-dilaton couplings. Similarly, we can define all dimensionful fields in n dimensions by properly dressing with the dilaton and the scale transformation properties follow from the Diff. In this sense, the mass dimension of a field is not necessarily the same as its scale dimension. For example, the dilaton has mass dimension $(n - 2)/2$, but its scale dimension is not even defined.

One can also easily check that the dilaton is, after all, a Lorentz scalar, hence so is $\Phi_{[d]}$. Thus, from the low energy point of view $\Phi_{[d]}$ and ϕ are indistinguishable from the usual scalar field. This clearly shows that the dilatations in Minkowski space can be derived from the Diff of virtual spacetime geometry of $g_{\mu\nu} = e^{2\kappa\phi}\eta_{\mu\nu}$ and we are never required to introduce Weyl symmetry.

From the supersymmetric point of view, the dilaton is inevitably associated with the axion so that the dilaton, axion and dilatino form a supermultiplet. In the dilaton geometry the axion can be easily incorporated by generalizing χ to include a phase such that eq.(2.9) is replaced by a new definition

$$\chi = e^{\kappa\phi_c}, \quad \kappa\phi_c \equiv \kappa\phi + i\frac{a}{f_a} \quad (2.14)$$

and

$$g_{\mu\nu} = \chi^*\chi\eta_{\mu\nu}. \quad (2.15)$$

Demanding that χ should still transform like a scale-dimension one field under dilatations, the axion transformation rule can be obtained:

$$\delta a = \alpha x^\mu \partial_\mu a. \quad (2.16)$$

The axion is a scale-dimensionless field. In general, under Diff the axion transforms like a scalar so that

$$\delta a = v^\mu \partial_\mu a. \quad (2.17)$$

To make sure this field is really entitled to be named as the axion, we can in fact check the relation between a and the (gravitational) axion associated with the antisymmetric field $B_{\mu\nu}$. In the dilaton geometry the relation between the axion and $B_{\mu\nu}$ is given by

$$\partial_\sigma a = \frac{1}{2} e^{-2\kappa} \epsilon_{\sigma\lambda\mu\nu} \partial^\lambda B^{\mu\nu}. \quad (2.18)$$

From χ we can derive

$$\partial_\sigma a = \frac{i}{2} e^{-2\kappa\phi} (\chi \partial_\sigma \chi^* - \chi^* \partial_\sigma \chi). \quad (2.19)$$

Thus, identifying

$$\epsilon_{\sigma\lambda\mu\nu} \partial^\lambda B^{\mu\nu} = i (\chi \partial_\sigma \chi^* - \chi^* \partial_\sigma \chi) \quad (2.20)$$

the axion in eq.(2.14) can indeed be identified as the standard axion. We shall also later see that this axion is in fact the R-axion and has the similar couplings as the axion associated with the Peccei-Quinn symmetry.

For complex χ the dressing of the ordinary fields can be generalized in an obvious way except for fermions. Fermions should be dressed over each Weyl components such as

$$\Psi_{[d]} \equiv \begin{pmatrix} \psi_{1[d]\alpha} \\ \frac{\psi_{2[d]\alpha}}{\chi^{\dot{\alpha}}} \end{pmatrix} = \begin{pmatrix} \chi^d & 0 \\ 0 & \chi^{*d} \end{pmatrix} \begin{pmatrix} \psi_{1\alpha} \\ \frac{\psi_{2\alpha}}{\chi^{\dot{\alpha}}} \end{pmatrix} \equiv \begin{pmatrix} \chi^d & 0 \\ 0 & \chi^{*d} \end{pmatrix} \Psi. \quad (2.21)$$

3. Superconformal Vector Fields and Superconformal Algebra

Before I generalize the dilaton geometry to the supersymmetric case, let us recapture the superconformal geometry. The superconformal geometry is described by superconformal vector fields which satisfy the superconformal algebra. The well known $N = 1$ superconformal algebra is given by the follows³.

³Our convention here follows that of [22].

First, the usual conformal algebra

$$[M_{\mu\nu}, M_{\rho\sigma}] = \eta_{\nu\rho}M_{\mu\sigma} - \eta_{\mu\rho}M_{\nu\sigma} - \eta_{\nu\sigma}M_{\mu\rho} + \eta_{\mu\sigma}M_{\nu\rho}, \quad (3.1)$$

$$[P_\mu, M_{\rho\sigma}] = \eta_{\mu\rho}P_\sigma - \eta_{\mu\sigma}P_\rho, \quad (3.2)$$

$$[K_\mu, M_{\rho\sigma}] = \eta_{\mu\rho}K_\sigma - \eta_{\mu\sigma}K_\rho, \quad (3.3)$$

$$[P_\mu, D_d] = P_\mu, \quad (3.4)$$

$$[K_\mu, D_d] = -K_\mu, \quad (3.5)$$

$$[P_\mu, K_\nu] = 2(\eta_{\mu\nu}D_d - M_{\mu\nu}), \quad (3.6)$$

and for the ordinary Poincaré supersymmetry

$$[Q, M_{\mu\nu}] = -\sigma_{\mu\nu}Q, \quad (3.7)$$

$$[Q, D_d] = \frac{1}{2}Q, \quad (3.8)$$

$$[Q, P_\mu] = 0, \quad (3.9)$$

$$[Q, K_\mu] = -\gamma_\mu S, \quad (3.10)$$

$$\{Q, \overline{Q}\} = 2i\gamma^\mu P_\mu. \quad (3.11)$$

The superconformal part requires an additional S-supersymmetry

$$[S, M_{\mu\nu}] = -\sigma_{\mu\nu}S, \quad (3.12)$$

$$[S, D_d] = -\frac{1}{2}S, \quad (3.13)$$

$$[S, P_\mu] = \gamma_\mu Q, \quad (3.14)$$

$$[S, K_\mu] = 0, \quad (3.15)$$

$$\{S, \overline{S}\} = 2i\gamma^\mu K_\mu. \quad (3.16)$$

The anti-commutator between Q- and S-supersymmetry

$$\{S, \overline{Q}\} = -2iD_d + 2i\sigma^{\mu\nu}M_{\mu\nu} - 3\gamma_5 R. \quad (3.17)$$

As is well known, this shows the R-symmetry is inevitably required in the superconformal symmetry. Finally, those involving R ,

$$[Q, R] = i\gamma_5 Q, \quad (3.18)$$

$$[S, R] = -i\gamma_5 S, \quad (3.19)$$

and all others vanish. We have written the algebra in terms of superconformal vector fields so that the operator algebra follows by $\mathcal{O} \rightarrow -i\mathcal{O}$ prescription for any operator \mathcal{O} in the above except Q, S . This leads to the correct eigenvalue problem for the hamiltonian $H = i\partial_0$.

The superconformal vector fields that satisfy the above superconformal algebra can be derived by solving the superconformal condition

$$[D_\alpha, \widehat{X}] = F_\alpha{}^\beta D_\beta, \quad (3.20)$$

where $F_\alpha{}^\beta$ is a function in superspace. \widehat{X} is a vector field in superspace such that

$$\widehat{X} = V^\mu \partial_\mu + \xi^\alpha \frac{\partial}{\partial \theta^\alpha} + \bar{\xi}_{\dot{\alpha}} \frac{\partial}{\partial \bar{\theta}_{\dot{\alpha}}}. \quad (3.21)$$

V^μ and ξ are determined by eq.(3.20) and we obtain

$$V^\mu = v^\mu + i \left(\theta \sigma^\mu \bar{\zeta} - \zeta \sigma^\mu \bar{\theta} \right) + \frac{1}{4} \theta \theta \left(\bar{\sigma}^\lambda \partial_\lambda \bar{\zeta} \sigma^\mu \bar{\theta} \right) + \bar{\theta} \bar{\theta} \left(\theta \sigma^\mu \bar{\sigma}^\lambda \partial_\lambda \right) + \frac{1}{4} \theta \theta \bar{\theta} \bar{\theta} \partial^\lambda \partial_\lambda v^\mu, \quad (3.22)$$

$$\xi^\alpha = \zeta^\alpha + \theta \sigma^\mu \bar{\sigma}^\nu f_{\mu\nu} + i \theta \sigma^\mu \bar{\theta} \partial_\mu \zeta^\alpha - \frac{i}{2} \theta \theta \bar{\sigma}^{\lambda\dot{\alpha}\alpha} \partial_\lambda \bar{\zeta}_{\dot{\alpha}} - \frac{i}{2} \theta \theta \bar{\theta}^{\dot{\gamma}} \epsilon^{\beta\gamma} \sigma_{\gamma\dot{\gamma}}^\lambda \sigma_{\beta\dot{\beta}}^\mu \bar{\sigma}^{\nu\dot{\beta}\alpha} \partial_\lambda f_{\mu\nu}, \quad (3.23)$$

and

$$\epsilon^{\mu\nu\sigma\lambda} \partial_\lambda f_{\text{AR}\sigma\nu} = \eta^{\lambda\nu} \partial_\mu f_{\text{SI}\lambda\nu} - 2\eta^{\mu\nu} \partial^\lambda f_{\text{AI}\lambda\nu}, \quad (3.24)$$

$$\partial_\mu v_\nu + \partial_\nu v_\mu = -4\eta_{\mu\nu} \eta^{\sigma\lambda} f_{\text{SR}\sigma\lambda}, \quad (3.25)$$

$$\partial_\mu v_\nu - \partial_\nu v_\mu = -8f_{\text{AR}\mu\nu} - 4\epsilon^{\sigma\lambda}_{\mu\nu} f_{\text{AI}\sigma\lambda}, \quad (3.26)$$

$$\partial^\lambda \partial_\lambda \zeta = 0, \quad (3.27)$$

for symmetric and antisymmetric components of real and imaginary parts of $f_{\mu\nu}$ defined by

$$f_{\mu\nu} \equiv f_{\text{SR}\mu\nu} + f_{\text{AR}\mu\nu} + i(f_{\text{SI}\mu\nu} + f_{\text{AI}\mu\nu}). \quad (3.28)$$

In fact, ξ is chiral so that in terms of $y^\mu \equiv x^\mu + i\theta \sigma^\mu \bar{\theta}$ it simplifies as

$$\xi^\alpha = \zeta^\alpha(y) + f_{\mu\nu}(y) \theta^\beta \sigma_{\beta\dot{\alpha}}^\mu \bar{\sigma}^{\nu\dot{\alpha}\alpha} + \theta \theta \kappa^\alpha(y), \quad (3.29)$$

$$\kappa^\alpha = -\frac{i}{2} \bar{\sigma}^{\lambda\dot{\alpha}\alpha} \partial_\lambda \bar{\zeta}_{\dot{\alpha}}. \quad (3.30)$$

Note that not all the components of $f_{\mu\nu}$ are uniquely determined by the superconformal condition. Eq.(3.25) leads to the usual conformal transformations and eq.(3.27) is the well known property of superconformal transformations, i.e. ζ is harmonic[23].

The detail of $F_\alpha{}^\beta$ is not important, but if we add for the sake of completeness, then

$$\begin{aligned} F_\alpha{}^\beta &= \sigma_{\alpha\dot{\alpha}}^\mu \bar{\sigma}^{\nu\dot{\alpha}\beta} f_{\mu\nu} + 2i\sigma_{\alpha\dot{\alpha}}^\mu \bar{\theta}^{\dot{\alpha}} \partial_\mu \zeta^\beta - i\theta_\alpha \bar{\sigma}^{\lambda\dot{\alpha}\beta} \partial_\lambda \bar{\zeta}_{\dot{\alpha}} - i \left(\theta_\alpha \bar{\theta}^{\dot{\alpha}} \epsilon^{\gamma\delta} \sigma_{\delta\dot{\alpha}}^\lambda + \theta^\gamma \bar{\theta}^{\dot{\alpha}} \sigma_{\alpha\dot{\alpha}}^\lambda \right) \sigma_{\gamma\dot{\beta}}^\mu \bar{\sigma}^{\nu\dot{\beta}\beta} \partial_\lambda f_{\mu\nu} \\ &\quad + \frac{1}{2} \theta^\gamma \bar{\theta}^{\dot{\alpha}} \epsilon^{\dot{\alpha}\dot{\gamma}} \sigma_{\alpha\dot{\alpha}}^\mu \sigma_{\gamma\dot{\gamma}}^\lambda \partial_\mu \partial_\lambda \zeta^\beta + \frac{1}{2} \theta \theta \bar{\theta}^{\dot{\alpha}} \sigma_{\alpha\dot{\alpha}}^\mu \bar{\sigma}^{\lambda\dot{\beta}\beta} \partial_\mu \partial_\lambda \bar{\zeta}_{\dot{\beta}} + \frac{1}{4} \theta \theta \bar{\theta} \bar{\theta} \sigma_{\alpha\dot{\alpha}}^\mu \bar{\sigma}^{\lambda\dot{\alpha}\delta} \sigma_{\delta\dot{\beta}}^\sigma \bar{\sigma}^{\nu\dot{\beta}\beta} \partial_\mu \partial_\lambda f_{\sigma\nu} \end{aligned} \quad (3.31)$$

Solving eqs.(3.24-3.27), we can obtain a representation of the generators for $N = 1$ superconformal algebra as⁴

$$M_{\mu\nu} = x_\mu \partial_\nu - x_\nu \partial_\mu - \left(\theta \sigma_{\mu\nu} \frac{\partial}{\partial \theta} + \bar{\theta} \bar{\sigma}_{\mu\nu} \frac{\partial}{\partial \bar{\theta}} \right), \quad (3.32)$$

$$P_\mu = \partial_\mu, \quad (3.33)$$

$$D_d = x^\mu \partial_\mu + \frac{1}{2} \left(\theta \frac{\partial}{\partial \theta} + \bar{\theta} \frac{\partial}{\partial \bar{\theta}} \right), \quad (3.34)$$

$$R = \theta \frac{\partial}{\partial \theta} - \bar{\theta} \frac{\partial}{\partial \bar{\theta}}, \quad (3.35)$$

$$K_\mu = \left(2x_\mu x^\lambda - x^2 \delta_\mu^\lambda - \theta \bar{\theta} \bar{\theta} \theta \delta_\mu^\lambda \right) \partial_\lambda - \left(x^\lambda \sigma_\mu \bar{\sigma}_\lambda \theta + 2i \theta \sigma_\mu \bar{\theta} \theta \right) \frac{\partial}{\partial \theta} - \left(x^\lambda \sigma_\mu \bar{\sigma}_\lambda \bar{\theta} - 2i \theta \sigma_\mu \bar{\theta} \bar{\theta} \right) \frac{\partial}{\partial \bar{\theta}}, \quad (3.36)$$

$$Q_\alpha = \frac{\partial}{\partial \theta^\alpha} - i \sigma_{\alpha\dot{\alpha}}^\lambda \bar{\theta}^{\dot{\alpha}} \partial_\lambda, \quad (3.37)$$

$$S_\alpha = - \left(i x^\lambda \sigma_{\lambda\alpha\dot{\beta}} \bar{\sigma}^{\mu\dot{\beta}\beta} \theta_\beta + \sigma_{\alpha\dot{\beta}}^\mu \bar{\theta}^{\dot{\beta}} \theta \theta \right) \partial_\mu - 4i \theta_\alpha \theta \frac{\partial}{\partial \theta} + \left(x^\lambda \sigma_{\lambda\alpha\dot{\beta}} + 2i \theta_\alpha \bar{\theta}_{\dot{\beta}} \right) \frac{\partial}{\partial \bar{\theta}_{\dot{\beta}}}. \quad (3.38)$$

4. Supersymmetric Dilaton Geometry

For supersymmetry we consider vielbeins e_μ^a and their superpartners, the gravitino, ψ_μ^α . Then the dilaton geometry of eq.(2.4) is given by

$$e_\mu^a = e^{\kappa\phi} \delta_\mu^a \quad (4.1)$$

for ϕ defined in eq.(2.4) and

$$\frac{1}{n} \partial_\lambda v^\lambda \delta_\mu^a = \delta_\lambda^a \partial_\mu v^\lambda. \quad (4.2)$$

The supersymmetric dilaton geometry needs fermionic analog of eq.(4.1) and the following is a good candidate⁵:

$$\psi_\mu^\alpha = \bar{\sigma}_\mu^{\dot{\alpha}\alpha} \bar{\psi}_{\dot{\alpha}}, \quad (4.3)$$

where σ -matrices are those in flat space in the same spirit as eq.(4.1).

To see if eq.(4.3) works we need to show that its supersymmetric Diff transformations correctly reproduces the superconformal transformations of flat superspace. Note that under supersymmetric Diff, ψ_μ^a picks up an extra fermionic term such that

$$\delta \psi_\mu^\alpha = v^\lambda \partial_\lambda \psi_\mu^\alpha + \partial_\mu v^\lambda \psi_\lambda^\alpha + 2 \partial_\mu \zeta^\alpha. \quad (4.4)$$

⁴We follow the same normalization convention of [23].

⁵We thank M. Grisaru for suggesting the gamma trace.

Then for eq.(4.3) we obtain

$$\bar{\sigma}_\mu^{\dot{\alpha}\alpha} \delta \bar{\psi}_{\dot{\alpha}} = v^\lambda \bar{\sigma}_\mu^{\dot{\alpha}\alpha} \partial_\lambda \bar{\psi}_{\dot{\alpha}} + \bar{\sigma}_\lambda^{\dot{\alpha}\alpha} \bar{\psi}_{\dot{\alpha}} \partial_\mu v^\lambda + 2 \partial_\mu \zeta^\alpha. \quad (4.5)$$

Contracting with flat σ^μ leads to

$$\delta \bar{\psi}_{\dot{\alpha}} = v^\mu \partial_\mu \bar{\psi}_{\dot{\alpha}} + \frac{1}{n} \bar{\psi}_{\dot{\alpha}} \partial_\mu v^\mu - \frac{2}{n} \sigma_{\alpha\dot{\alpha}}^\mu \partial_\mu \zeta^\alpha. \quad (4.6)$$

Consistency conditions between eqs.(4.5-4.6) can be obtained by plugging eq.(4.6) back into eq.(4.5) and they read

$$-\frac{2}{n} \bar{\sigma}_\mu^{\dot{\alpha}\alpha} \sigma_{\beta\dot{\alpha}}^\lambda \partial_\lambda \zeta^\beta = 2 \partial_\mu \zeta^\alpha. \quad (4.7)$$

This is the superconformal condition in flat space, the counter-part of eq.(4.2). Now it is easy to show that this is indeed equivalent to the well-known superconformal conditions. Solutions to eq.(4.7) can be written as

$$\partial_\mu \zeta^\alpha = \bar{\sigma}_\mu^{\dot{\alpha}\alpha} \bar{\eta}_{\dot{\alpha}}, \quad (4.8)$$

and eq.(4.7) implies, for $n \neq 1$,

$$2 \bar{\sigma}_\mu^{\dot{\alpha}\alpha} \partial^\mu \bar{\eta}_{\dot{\alpha}} = 2 \partial^\mu \partial_\mu \zeta^\alpha = \frac{2}{n} \partial^\mu \partial_\mu \zeta^\alpha = 0. \quad (4.9)$$

Therefore, ζ^α is harmonic and $\bar{\eta}_{\dot{\alpha}}$ is a constant spinor and that we have recovered the well known superconformal conditions in flat superspace[23].

In the above sense, eqs.(4.1,4.3) define the supersymmetric dilaton geometry and we identify ϕ as the dilaton and ψ as its superpartner, the dilatino. (In practice, we shall later identify a scaled one as the dilatino. See the next section.) Their transformation rules under dilatations are given by eq.(2.5) and eq.(4.6) respectively. Thus the supersymmetric dilaton geometry is an effective way of describing the supersymmetric dilatations in the flat space. For the dilaton and dilatino to form a supermultiplet we must include the axion, hence defining the dilaton-axion chiral supermultiplet, (ϕ_c, ψ) . The R-charge of the dilaton-axion multiplet can be easily determined from eq.(2.16) and the superconformal symmetry such that its R-charge is zero.

5. Effective Lagrangian

We demand the superconformally invariant lagrangian to be covariant in the supersymmetric dilaton geometry. Thus, such a scale invariant effective lagrangian for the dilaton and the dilatino can be read off from the supergravity lagrangian[24][22] using the metric of the dilaton geometry. For simplicity, let us consider the $N = 1$ supergravity lagrangian in four dimensions

$$\mathcal{L}_{\text{S.G.}} = -\frac{1}{2\kappa^2} e R(e, \Omega) + \frac{1}{2} e \tilde{\epsilon}^{\lambda\sigma\mu\nu} \left(\bar{\psi}_\lambda \bar{\sigma}_\sigma \mathcal{D}_\mu \psi_\nu - \psi_\lambda \tilde{\sigma}_\sigma \mathcal{D}_\mu \bar{\psi}_\nu \right), \quad (5.1)$$

where $\tilde{\sigma}_\sigma$ and $\tilde{\epsilon}^{\lambda\sigma\mu\nu}$ are in curved space and $\mathcal{D}_\mu = \partial_\mu + \frac{1}{2}\Omega_\mu^{ab}\sigma_{ab}$. The spin connection also contains the contribution from the gravitino. The axion kinetic energy term can be easily included as the antisymmetric tensor term, i.e. H^2 term.

In the dilaton geometry the spin connection is given by

$$\begin{aligned}\Omega_{\mu a}^{b} &\equiv \omega_{\mu a}^{b} + \kappa_{\mu a}^{b}, \\ \omega_{\mu a}^{b} &= \kappa \left(\delta_\mu^b \delta_a^\lambda - \eta^{b\lambda} \eta_{\mu a} \right) \partial_\lambda \phi, \\ \kappa_{\mu a}^{b} &= -\frac{\kappa^2}{2} e^{-\kappa\phi} \epsilon_{\mu a}^{bc} \bar{\psi} \bar{\sigma}_c \psi\end{aligned}\tag{5.2}$$

so that we obtain

$$eR(e, \Omega) = 6\kappa^2 e^{2\kappa\phi} \eta^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + 3\kappa^4 \bar{\psi} \bar{\psi} \psi \psi + \dots,\tag{5.3}$$

where the ellipsis is a total derivative. The gravitino term leads to

$$\frac{1}{2} e \tilde{\epsilon}^{\lambda\sigma\mu\nu} \left(\bar{\psi}_\lambda \tilde{\sigma}_\sigma \mathcal{D}_\mu \psi_\nu - \psi_\lambda \tilde{\sigma}_\sigma \mathcal{D}_\mu \bar{\psi}_\nu \right) = -3ie^{\kappa\phi} \left(\bar{\psi} \bar{\sigma}^\mu \partial_\mu \psi + \psi \sigma^\mu \partial_\mu \bar{\psi} \right).\tag{5.4}$$

This is almost the dilatino kinetic energy term except the prefactor involving the dilaton. This prefactor can be removed easily by scaling as

$$\psi \equiv e^{-\frac{1}{2}(\kappa\phi + i\frac{a}{f_a})} \lambda,\tag{5.5}$$

then the kinetic term leads to supersymmetric dilaton effective lagrangian

$$\mathcal{L}_{\text{dil}} = -\frac{1}{2} e^{2\kappa\phi} \eta^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} e^{2\kappa\phi} \eta^{\mu\nu} \partial_\mu a \partial_\nu a - \frac{1}{2} i \left(\bar{\lambda} \bar{\sigma}^\mu \mathcal{D}_\mu \lambda + \lambda \sigma^\mu \mathcal{D}_\mu \bar{\lambda} \right) - \frac{\kappa^2}{4} e^{-2\kappa\phi} \bar{\lambda} \bar{\lambda} \lambda \lambda,\tag{5.6}$$

where $\mathcal{D}_\mu = \partial_\mu - \frac{i}{2f_a} \partial_\mu a$ and the dilatino λ is a Majorana-Weyl fermion. Note that the dilaton-dilatino coupling shows that this is a four dimensional analog to the two-dimensional super-Liouville theory lagrangian (if the axion terms are dropped). In two dimensions the corresponding term is renormalizable, but not in four dimensions. Perhaps, this could be the reason why this lagrangian has never been investigated. Note that in this field configuration the prefactor of the four-dilatino term is $e^{-2\kappa\phi}$, not $e^{+2\kappa\phi}$.

By construction, this effective lagrangian is invariant under superconformal transformations:

$$\delta e^{\kappa\phi} = \frac{1}{n} e^{\kappa\phi} D_\mu v^\mu,\tag{5.7}$$

$$\delta a = v^\mu \partial_\mu a,\tag{5.8}$$

$$\delta \lambda = \frac{3}{8} \lambda \partial_\mu v^\mu + v^\mu \partial_\mu \lambda - \frac{1}{2} e^{\frac{1}{2}(\kappa\phi + i\frac{a}{f_a})} \sigma^\mu \partial_\mu \bar{\zeta}.\tag{5.9}$$

Therefore, in particular, it is scale invariant.

There is another global supersymmetry that leaves the lagrangian invariant:

$$\delta_\epsilon \phi = 2(\epsilon\lambda + \bar{\epsilon}\bar{\lambda}), \quad (5.10)$$

$$\delta_\epsilon a = -2i(\epsilon\lambda - \bar{\epsilon}\bar{\lambda}), \quad (5.11)$$

$$\delta_\epsilon \lambda = \kappa(\epsilon\lambda + \bar{\epsilon}\bar{\lambda})\lambda + \frac{1}{f_a}(\epsilon\lambda - \bar{\epsilon}\bar{\lambda})\lambda + \frac{5}{8}\kappa(\bar{\epsilon}\bar{\lambda})\lambda + \frac{i}{4}e^{2\kappa\phi}\sigma^\mu\mathcal{D}_\mu\phi\bar{\epsilon}. \quad (5.12)$$

As a matter of fact, this is not the only global supersymmetry we can obtain but we have chosen it such that $\delta_\epsilon \phi$ does not involve ϕ in the RHS of eq.(5.10). Since we have already got rid of auxiliary fields implicitly, the proof of this global supersymmetry requires the equations of motion

$$0 = \partial_\mu \partial^\mu \phi + \kappa \partial_\mu \phi \partial^\mu \phi - \kappa \partial_\mu a \partial^\mu a + \frac{\kappa^3}{2}e^{-4\kappa\phi}\bar{\lambda}\lambda\lambda, \quad (5.13)$$

$$0 = \partial^\mu \left(e^{2\kappa\phi} \partial_\mu a - \frac{i}{2f_a} \bar{\lambda} \bar{\sigma}_\mu \lambda \right), \quad (5.14)$$

$$0 = i\bar{\sigma}^\mu \mathcal{D}_\mu \lambda + \frac{\kappa^2}{4}e^{-2\kappa\phi}\lambda\lambda\bar{\lambda}. \quad (5.15)$$

Next, let us consider couplings of chiral supermultiplets (z^i, ψ^i) and vector supermultiplets (A_μ^a, λ^a) . The Kähler and super-potentials are now $K(\phi_c, \bar{\phi}_c, z^i, \bar{z}^i)$ and $W(\phi_c, z^i)$, where z^i are now dilaton dressed fields and ϕ_c is given in eq.(2.14). We choose $K = z^i \bar{z}^i$ (before dressing) so that the Kähler metric is $g_{ij} = \delta_{ij}$. Then the matter part is given by

$$\mathcal{L}_{\text{matter}} = \mathcal{L}_1 + \mathcal{L}_2 - V, \quad (5.16)$$

where

$$\begin{aligned} \mathcal{L}_1 = & -\mathcal{D}_\mu^{(1)} z^i \bar{\mathcal{D}}^{(1)\mu} \bar{z}^i - i\bar{\psi}^i \bar{\sigma}^\mu \tilde{\mathcal{D}}_\mu \psi^i - \frac{1}{16\pi} \tau^I F_{\mu\nu}^a F^{a\mu\nu} + \frac{1}{32\pi} \tau^R \epsilon^{\mu\nu\lambda\sigma} F_{\mu\nu}^a F_{\lambda\sigma}^a \\ & - i\bar{\lambda}^a \bar{\sigma}^\mu \tilde{\mathcal{D}}_\mu \lambda^a + \left(i\sqrt{2} g e^{-i\frac{2}{f_a}a} T_{ij}^a z^j \psi^i \lambda^a + \text{h.c.} \right) \end{aligned} \quad (5.17)$$

$$\begin{aligned} \mathcal{L}_2 = & \kappa e^{-\kappa\phi} \left(2g e^{i\frac{2}{f_a}a} D^{(a)} \bar{\lambda} \lambda^a - \sqrt{2} \bar{\mathcal{D}}_\mu^{(1)} \bar{z}^i \psi^i \sigma^\mu \bar{\lambda} + \text{h.c.} \right) \\ & + \kappa^2 e^{-2\kappa\phi} \left[\frac{1}{2} (\bar{\psi}^i \bar{\lambda}) (\psi^i \lambda) + \frac{i}{4} \left(\bar{z}^i \mathcal{D}_\mu^{(1)} z^i - z^i \bar{\mathcal{D}}_\mu^{(1)} \bar{z}^i \right) \bar{\psi}^j \bar{\sigma}^\mu \psi^j - \frac{3}{2} (\bar{\lambda}^a \bar{\lambda}) (\lambda^a \lambda) \right. \\ & \left. + \frac{i}{4} \left(\bar{z}^i \mathcal{D}_\mu^{(1)} z^i - z^i \bar{\mathcal{D}}_\mu^{(1)} \bar{z}^i \right) \bar{\lambda}^a \bar{\sigma}^\mu \lambda^a - \frac{1}{16\pi} \tau^I (\bar{\psi}^i \bar{\lambda}^a) (\psi^i \lambda^a) + \frac{3}{8} (\bar{\lambda}^a \bar{\lambda}^b) (\lambda^a \lambda^b) \right] \\ & - \exp \left(\frac{1}{2} \kappa^2 z^k \bar{z}^k e^{-2\kappa\phi} \right) \left[6\kappa^2 e^{-2\kappa\phi - i\frac{4}{f_a}a} \bar{W} \bar{\lambda} \bar{\lambda} - i2\sqrt{2}\kappa e^{-\kappa\phi - i\frac{4}{f_a}a} D_i W \psi^i \lambda \right. \\ & \left. + \frac{1}{2} e^{-i\frac{4}{f_a}a} \psi^i \psi^j \left(\partial_i \partial_j W + 2\kappa^2 e^{-2\kappa\phi} \bar{z}_i D_j W - \kappa^4 e^{-4\kappa\phi} \bar{z}_i \bar{z}_j W \right) + \text{h.c.} \right] \end{aligned} \quad (5.18)$$

$$V = V_F + V_D, \quad (5.19)$$

$$V_F \equiv \exp \left(\kappa^2 z^i \bar{z}^i e^{-2\kappa\phi} \right) \left(D_j W \bar{D}_j \bar{W} - 3\kappa^2 e^{-2\kappa\phi} W \bar{W} \right),$$

$$V_D \equiv \frac{1}{2} g^2 D^{(a)} D^{(a)}.$$

In the above, we used $\tau \equiv \frac{\theta}{2\pi} + i\frac{4\pi}{g^2}$, $D^{(a)} \equiv \bar{z}^i T_{ij}^a z^j$, $\mathcal{D}_\mu^{(k)} \equiv \partial_\mu - k(\kappa\partial_\mu\phi + \frac{i}{f_a}\partial_\mu a) + igA_\mu^a T^a$, $\tilde{\mathcal{D}}_\mu \equiv \mathcal{D}_\mu^{(0)} - i\frac{3}{2f_a}\partial_\mu a$, $\partial_i \equiv \partial/\partial z^i$ and $D_i W \equiv \partial_i W + \kappa^2 e^{-2\kappa\phi} \bar{z}_i W$. Note that in the fermionic kinetic energy terms there are no dilaton contributions. This is because the spin connection contribution from the dilaton geometry exactly cancels the term from the dilaton dressing. This lagrangian is superconformally invariant classically by construction due to the supergravity of the supersymmetric dilaton geometry. The superconformal transformation rules for matter fields are the standard ones and straightforward.

The full lagrangian also has a global supersymmetry

$$\delta_\epsilon \phi = 2(\epsilon\lambda + \bar{\epsilon}\bar{\lambda}), \quad (5.20)$$

$$\delta_\epsilon a = -2i(\epsilon\lambda - \bar{\epsilon}\bar{\lambda}), \quad (5.21)$$

$$\begin{aligned} \delta_\epsilon \lambda = & \frac{1}{2} \left(\kappa\delta_\epsilon \phi + \frac{i}{f_a}\delta_\epsilon a \right) \lambda + \frac{5}{8}\kappa(\bar{\epsilon}\bar{\lambda})\lambda + \frac{i}{4}e^{2\kappa\phi}\sigma^\mu\mathcal{D}_\mu\phi\bar{\epsilon} + \frac{i}{16} \left(\bar{z}^i\sigma^\mu\mathcal{D}_\mu^{(1)}z^i - z^i\sigma^\mu\bar{\mathcal{D}}_\mu^{(1)}\bar{z}^i \right) \bar{\epsilon} \\ & + \frac{3}{8}\kappa(\bar{\epsilon}\bar{\psi}^i)\psi^i - \frac{5}{8}\kappa(\bar{\epsilon}\bar{\lambda}^a)\lambda^a + \frac{i}{2\sqrt{2}}\kappa^2 e^{-\kappa\phi} \left(z^i\bar{\epsilon}\bar{\psi}^i - \bar{z}^i\epsilon\psi^i \right) \lambda + \exp\left(\frac{1}{2}\kappa^2 z^i\bar{z}^i e^{-2\kappa\phi}\right) \bar{W}\epsilon, \end{aligned} \quad (5.22)$$

$$\delta_\epsilon z^i = i\sqrt{2}\epsilon\psi^i e^{\kappa\phi} + \left(\kappa\delta_\epsilon \phi + \frac{i}{f_a}\delta_\epsilon a \right) z^i, \quad (5.23)$$

$$\begin{aligned} \delta_\epsilon \psi^i = & \frac{3}{2} \left(\kappa\delta_\epsilon \phi + \frac{i}{f_a}\delta_\epsilon a \right) \psi^i - \kappa(\bar{\epsilon}\bar{\lambda})\psi^i + \sqrt{2}e^{\kappa\phi+i\frac{a}{f_a}}\sigma^\mu\mathcal{D}_\mu^{(1)}z^i\bar{\epsilon} + \frac{i}{2\sqrt{2}}\kappa^2 e^{-\kappa\phi-i\frac{a}{f_a}} \left(z^j\bar{\epsilon}\bar{\psi}^j - \bar{z}^j\epsilon\psi^j \right) \psi^i \\ & - i\sqrt{2}\exp\left(\frac{1}{2}\kappa^2 z^k\bar{z}^k e^{-2\kappa\phi}\right) e^{\kappa\phi+i4\frac{a}{f_a}}\bar{D}^i\bar{W}\epsilon, \end{aligned} \quad (5.24)$$

$$\delta_\epsilon A_\mu^a = -e^{\kappa\phi} \left(e^{i\frac{2}{f_a}a}\epsilon\sigma_\mu\bar{\lambda}^a - e^{-i\frac{2}{f_a}a}\bar{\epsilon}\sigma_\mu\lambda^a \right), \quad (5.25)$$

$$\delta_\epsilon \lambda^a = ie^{\kappa\phi} e^{i\frac{2}{f_a}a} \left(F_{\mu\nu}^a \sigma^{\mu\nu} - igD^{(a)} \right) \epsilon + \frac{3}{2} \left(\kappa\delta_\epsilon \phi + \frac{i}{f_a}\delta_\epsilon a \right) \lambda^a - \frac{i}{2\sqrt{2}}\kappa^2 e^{-\kappa\phi} \left(z^i\bar{\epsilon}\bar{\psi}^i + \bar{z}^i\epsilon\psi^i \right) \lambda^a. \quad (5.26)$$

The dilaton, axion and dilatino form additional chiral supermultiplet. The dilaton and axion couplings are unavoidably nonrenormalizable. As $\kappa \rightarrow 0$ and $f_a \rightarrow \infty$, $\mathcal{L}_2 \rightarrow -\frac{1}{2}\psi^i\psi^j\partial_i\partial_j W$ and $\mathcal{L}_{\text{matter}}$ reduces to the $N = 1$ supersymmetric gauge theory. Thus we can regard $\mathcal{L}_{\text{dil}} + \mathcal{L}_{\text{matter}}$ as the scale invariant generalization of $N = 1$ supersymmetric gauge theory incorporating the dilaton chiral supermultiplet. All the nonrenormalizable terms are dictated by the supersymmetric dilaton geometry.

To be consistent with the global supersymmetry the superpotential W must lead to a positive semidefinite scalar potential V for a superconformally invariant vacuum. Thus W cannot be arbitrary, but it is not difficult to construct examples. In general, however, it is not necessary to satisfy $V_F = |\partial_a \tilde{W}|^2$ for some superpotential (a now runs over the dilaton too) because there could be soft breaking terms due to the dilaton much analogous to the supergravity. It is known in the supergravity that the spontaneous breaking of local supersymmetry manifests itself as soft breaking of global supersymmetry[25]. In the case of unbroken supersymmetry with broken superconformal symmetry, the effective scalar potential might be expressed as $V_F = |\partial_a \tilde{W}|^2$,

where \widetilde{W} is not the same as W for an obvious reason. A similar line of thought also appears in softly broken $N = 2$ supersymmetric QCD[26]. I shall discuss more details in the following section.

6. Symmetry Breaking

In the supersymmetric case breaking scale symmetry is nothing but breaking the superconformal symmetry so that it automatically addresses the issue of supersymmetry breaking. The conventional wisdom in the absence of the dilaton is that the scale and R symmetries are anomalous, yet these anomalies leave the Poincaré supersymmetry unbroken.

In the presence of the dilaton we presume that the scale and R symmetries should be spontaneously broken. Thus we need a nonanomalous dilaton sector in which scale symmetry is spontaneously broken. In fact, this can also be done at nontrivial fixed points, where β -functions vanish, so that we can prevent the trace anomaly from causing any complications. The violation of the conservation of the scale current consists of two terms: the dilaton mass term and the trace anomaly. This is much analogous to the pion case of PCAC, in which the conservation of the axial current is violated by two terms: pion mass term and the axial anomaly. Looking at the superconformal algebra, we can easily understand that, if scale symmetry is spontaneously broken, the Poincaré supersymmetry could also be broken, unless the R-symmetry breaking is related to the scale symmetry breaking in a specific way. Furthermore, explicit soft supersymmetry breaking can occur too. This makes the situation much more complicated than the bosonic case.

Let us first consider the consequence of unbroken superconformal symmetry, hence, unbroken Poincaré supersymmetry. In this case, for the Lorentz invariant vacuum, eq.(3.17) implies

$$\langle 0|D_d|0\rangle = \frac{3}{2}i\langle 0|\gamma_5 R|0\rangle. \quad (6.1)$$

and in particular the vacuum is also an eigenstate for both D_d and R . Due to the presence of γ_5 , for nonvanishing vacuum expectation values, this identity can only be satisfied if the vacuum is fermionic such that

$$|0\rangle = \begin{pmatrix} |+\rangle \\ |-\rangle \end{pmatrix}. \quad (6.2)$$

The vacuum states are eigenstates of D_d or R , although other states are not. In particular, $|0\rangle$ need to be a Majorana spinor so that

$$R|\pm\rangle = \pm c|\pm\rangle. \quad (6.3)$$

Then for unbroken supersymmetry

$$D_d|\pm\rangle = \frac{3}{2}c|\pm\rangle. \quad (6.4)$$

Hence, for unbroken Poincaré supersymmetry the R -charge of the vacuum must be specifically related to its scale dimension[27]. Since the invariant vacuum should not carry a scale-dimension, we can consistently choose $c = 0$. Next, for a bosonic vacuum, eq.(6.1) is true only if

$$\langle 0|D_d|0\rangle = 0 = \langle 0|R|0\rangle. \quad (6.5)$$

The bosonic supersymmetric vacuum cannot carry a scale-dimension or R -charge. Therefore, for superconformally invariant vacuum we can always choose $D_d|0\rangle = 0 = R|0\rangle$.

This suggests us that we can use $D_d|0\rangle \neq 0$ ($R|0\rangle \neq 0$) as an indication for spontaneous breaking of scale symmetry (R -symmetry, respectively), hence, spontaneous breaking of conformal symmetry. This is nothing unusual for the R -symmetry, which is, more or less, internal. For the scale symmetry this is true only if it is not realized in the Wigner-Weyl way. For example, the translational symmetry is not broken even if the vacuum is not translationally invariant, which is the case of nonvanishing vacuum energy. This is because one can still find a unitary operator that leaves the Hilbert space invariant under the translation. So we have no violation of energy-momentum conservation despite the nonvanishing vacuum energy. Without the dilaton, equally one can argue this is the case. But, since the dilaton does not transform as a quasiprimary field under dilatations, the scale symmetry is not realized in the Wigner-Weyl way in the presence of the dilaton. Therefore, we can use $D_d|0\rangle \neq 0$ as the symmetry breaking condition. One can easily check that this is also consistent with the conservation law of the scale current much the same way as in any other internal symmetry cases.

If supersymmetry is broken spontaneously, the vacuum states are no longer eigenstates of D_d because H and D_d do not commute (this is the case for soft breaking too). For $E_0 = \langle 0|H|0\rangle \neq 0$ the vacuum seems to violate $[H, D_d] = iH$ because $\langle 0|[H, D_d]|0\rangle = 0$. In fact, this happens for any state with nonzero energy eigenvalue if we define $H|E\rangle = E|E\rangle$ with $\langle E|E\rangle = 1$ naively. This normalization condition is true only if the spectrum is discrete. The matter of the fact is that the conformal symmetry demands the spectrum to be continuous⁶, if $E \neq 0$. Therefore, the naive normalization condition is not correct, but should be replaced by $\langle E|E'\rangle = \delta(E - E')$. In particular, for continuous spectrum let $E_a = e^a E_0$, $a \geq 0$, then $\langle E_b|E_a\rangle = \frac{1}{E_a}\delta(b - a)$. Now $E_0 = \int dE_b \langle E_b|H|0\rangle = -i \int dE_b \langle E_b|[H, D_d]|0\rangle$, provided that $\langle E_b|D_d|E_a\rangle = -i \frac{1}{\sqrt{E_b E_a}} \frac{\partial}{\partial b} \delta(b - a)$.

⁶The author thanks D.Z. Freedman for pointing out this.

In the superconformal case, $[R, H] = 0 = [D_d, R]$ implies the energy eigenstate can be written as $|E, r\rangle = \sum_d a_d(E) |d, r\rangle$, where $D_d |d, r\rangle = d |d, r\rangle$. Then

$$\langle E_b | D_d | E_a \rangle = \sum_d d a_d^*(E_b) a_d(E_a) = -i \frac{1}{\sqrt{E_b E_a}} \frac{\partial}{\partial b} \delta(b - a). \quad (6.6)$$

The normalization condition in turn requires

$$\sum_d a_d^*(E_b) a_d(E_a) = \frac{1}{E_a} \delta(b - a) \quad (6.7)$$

To be consistent, such $a_d(E)$ must exist in superconformal field theories and the above indeed yield a solution

$$a_d(E_a) = \frac{1}{\sqrt{2\pi E_a}} e^{-iad}. \quad (6.8)$$

Then, the continuous spectrum can be constructed in terms of $|E_a\rangle = e^{-\frac{1}{2}a} e^{-iaD} |E_0\rangle$ for any constant $a \geq 0$. In nonsupersymmetric case, despite the absence of $[R, D_d] = 0$, a similar construction is possible as long as the spectrum is positive definite. Thus energy eigenstates of nonvanishing energy eigenvalues are superpositions of dilatation eigenstates.

Notice that we can in fact construct states $|E_a\rangle$ for $a < 0$. Recall that in the above $a \geq 0$ condition is required simply due to the assumption that $|E_0\rangle$ is the vacuum $|0\rangle$. This implies if the conformal symmetry is unbroken, the only possible vacuum has to be $E_0 = 0$ to be consistent, and that $D_d |0\rangle = 0$. Hence, if supersymmetry is broken with nonzero vacuum energy, the conformal symmetry must be broken at the same time to make sure the states below the vacuum decoupled.

Next, let us ask if the superconformal symmetry can be broken without breaking the Poincaré supersymmetry. If $Q|0\rangle = 0$ and $D_d |0\rangle \neq 0$ (or $R|0\rangle \neq 0$) can be consistently satisfied, this can happen. For unbroken supersymmetry the vacuum must carry a specific R-charge and scaling dimension. A supersymmetric vacuum is Poincaré invariant so that the superconformal algebra forces the vacuum to be a zero mode of S . Therefore, $S|0\rangle = 0$ implies $R|0\rangle \neq 0$ and the vacuum must be fermionic such that $(D_d - i(3/2)\gamma_5 R)|0\rangle = 0$. Thus R-symmetry is also broken and that the scale symmetry breaking is specifically related to the R-symmetry breaking.

This indicates that, if the scale symmetry is spontaneously broken in the bosonic vacuum, the Poincaré supersymmetry must be broken as well as the superconformal symmetry. This can be a new way of breaking the supersymmetry using the dilaton. It implies that all of their symmetry breaking scales must be the same to be consistent. This puts a very severe constraint on the possibility of spontaneous superconformal symmetry breaking.

To achieve spontaneous breaking of the scale symmetry in the dilaton sector, we first need to take care of the issue of the trace anomaly. If there is a trace anomaly in the dilaton sector, then the scale symmetry is explicitly broken and the existence of the dilaton is meaningless. The pure dilaton sector only allows a specific form of potential which is unavoidably nonrenormalizable to be conformally invariant. Such a potential always puts the vacuum at infinity.

The remedy in nonsupersymmetric case is, as noted in [18], introducing a dynamical scale which transforms under dilatations as

$$\delta M = \alpha M, \quad (6.9)$$

we can in fact write down a dilaton effective potential which is free from a trace anomaly:

$$V_{\phi,\text{eff}} = \frac{\Lambda}{4} e^{4\kappa\phi} + \frac{1}{64\pi^2} \left(4\kappa^2 \Lambda e^{2\kappa\phi} \right)^2 \left(\log \frac{4\kappa^2 \Lambda e^{2\kappa\phi}}{M^2} - \frac{3}{2} \right). \quad (6.10)$$

One can easily check that this effective potential is scale invariant incorporating eq.(6.9). Without eq.(6.9), the $\phi e^{4\kappa\phi}$ term is not scale invariant. Note that introducing such a dynamical scale does not change trace anomalies in other sectors because it does not modify properties of β -functions. M can be simply used as the scale which enters in the renormalization group equation. Also there is no spontaneous breaking of scale symmetry in other sectors. The basic difference lies on the fact that the dilaton is not a quasiprimary field, whilst all other fields are quasiprimary fields.

This effective dilaton potential indeed admits a new vacuum in which the dilaton gets a nontrivial vacuum expectation value and becomes massive. The new minimum located at

$$\langle \phi \rangle = \frac{1}{2\kappa} \left(1 - \frac{\pi^2}{\kappa^4 \Lambda} + \log \frac{M^2}{4\kappa^2 \Lambda} \right) \quad (6.11)$$

for any $\kappa^4 \Lambda$. This new vacuum is no longer invariant under dilatations so that the scale symmetry is spontaneously broken. The conservation law of the total dilatation current in general will be violated by two terms as in the PCAC case of the pions:

$$\partial_\mu S^\mu = -\frac{m_\phi^2}{\kappa} \phi + \text{trace anomaly} \quad (6.12)$$

for small ϕ , where for the potential eq.(6.10) the dilaton mass is given by

$$m_\phi^2 = \frac{\kappa^2 M^4}{8\pi^2} \exp \left\{ 2 \left(1 - \frac{\pi^2}{\kappa^4 \Lambda} \right) \right\}. \quad (6.13)$$

Note that this dilaton mass is precisely the one arises in $V_{\phi,\text{eff}}$ after shifting the vacuum as $\phi \rightarrow \phi + \langle \phi \rangle$, hence confirming that our notion of spontaneous scale symmetry breaking is consistent. Once we obtain a vacuum with broken scale symmetry, M can be fixed to a numerical value. Thus, the dilaton mass depends on still-undetermined three parameters.

Eq.(6.10) cannot be obtained in the supersymmetric case because the corresponding superpotential must contain $\sqrt{\phi}$ term to lead to the given scalar potential. Nevertheless, the same idea can be applied and superpotentials that lead to spontaneous breaking of scale symmetry can be constructed. A useful form of the pure dilaton part of a scalar potential which exhibits spontaneous breaking of the scale symmetry in the same way as the bosonic case is

$$V_{\text{dil,eff}} = \kappa^2 \Lambda e^{4\kappa\phi} (\phi - a_1)(\phi - a_2), \quad (6.14)$$

where

$$\kappa a_i = \frac{1}{2} \ln \frac{\kappa^2 \Lambda}{M^2} + c_i, \quad i = 1, 2, \quad (6.15)$$

and c_i 's are constants independent of M . The specific forms of a_i are dictated to make the potential scale invariant so that there is no trace anomaly from this potential. This scalar potential can be derived from a superpotential, for example, with one scalar field, z , other than the dilaton,

$$W_{\text{eff}}(\phi_c, z) = \frac{\sqrt{\Lambda}}{2\sqrt{3}} (a_1 - a_2) e^{3\kappa\phi_c} + \sqrt{\Lambda} \kappa z e^{2\kappa\phi_c} \left(\phi_c - \frac{1}{2}(a_1 + a_2) \right). \quad (6.16)$$

The first term is an R-symmetry breaking term that vanishes as $(a_1 - a_2) \rightarrow 0$. $V_{\text{dil,eff}}$ has a scale symmetry breaking vacuum at

$$\langle \phi \rangle = v \equiv \frac{1}{2} \left(a_1 + a_2 - \frac{1}{2\kappa} + \sqrt{(a_1 - a_2)^2 + \frac{1}{4\kappa^2}} \right) \quad (6.17)$$

with $V_{\text{dil,eff}}(\langle \phi \rangle) \leq 0$. The equality is only for $a_1 = a_2$. The dilaton mass can be easily computed after shifting the vacuum and reads

$$m_\phi^2 = 2\kappa^2 \Lambda e^{4\kappa v} (1 + 2\kappa(2v - a_1 - a_2)). \quad (6.18)$$

As $v \rightarrow -\infty$, $m_\phi \rightarrow 0$, confirming the dilaton is massless in the scale invariant asymptotic vacuum.

To be more precise, the vacuum must be a vacuum for V , not just the pure dilaton part V_{dil} . The vacuum structure of V is fairly complicated even with just one chiral multiplet. It

includes vacua for both $V = 0$ and $V \neq 0$. For one chiral multiplet the vacuum we obtained for V_{dil} is not stable along the z direction if $a_1 \neq a_2$. Since the potential is bounded below, there must be another vacuum nearby that satisfies $V_F < 0$ and $\frac{\partial V}{\partial z} = 0 = \frac{\partial V}{\partial \phi_c}$. However, we expect either the scale symmetry must be also explicitly broken so that states with negative vacuum states can be allowed with potential bounded below, or $V = V_F + V_D \geq 0$ due to the D-term contribution. If not, this is not a desirable result because we want $V \geq 0$. Hence we may need to include more than one chiral multiplet.

If $a_1 = a_2 \equiv v$, then the vacuum is stable along the z direction too. In this case the vacuum energy also vanishes. One can also easily check that the supersymmetry is unbroken in this vacuum, although the scale symmetry is spontaneously broken with $\langle \phi \rangle = v$. The dilaton mass in this case is $m_\phi^2 = 2\kappa^2 \Lambda e^{4\kappa v}$. If we assume $c_i = 0$, then $m_\phi^2 = 2\kappa^4 \Lambda^2 / M^2$. This is an example that the scale symmetry breaking does not lead to broken supersymmetry as we analyzed before using the algebraic structure.

If $a_1 = a_2$, there is no axion contribution to the scalar potential in my examples. Otherwise, however, the scalar potential explicitly contains the axion potential in some cases. The axion potential roughly takes the form of $f(\phi, z, \bar{z})(a_1 - a_2)(g(\phi)z \cos \frac{a}{f_a} + \bar{z} \frac{a}{f_a} \sin \frac{a}{f_a})$ for smooth functions f and g . In particular, the potential is not periodic for the axion if $a \neq 0$. Anyhow, it still yields extrema at $a = 0$. This nonperiodicity for $a_1 \neq a_2$ is because the corresponding term in the superpotential does not have R-symmetry. Recall that the dilaton multiplet has R-charge zero and z has R-charge two. All the vacua I obtain determine $\langle a \rangle = 0$ if $a_1 \neq a_2$.

Next, let us ask if there is a vacuum with a vanishing vacuum energy and broken supersymmetry. For one chiral multiplet case we can find a solution, taking advantage of the Polonyi solution[28]. If we assume

$$\frac{1}{2\sqrt{3}} \frac{a_1 - a_2}{\phi_c - \frac{1}{2}(a_1 + a_2)} e^{\kappa\phi_c} = \pm(2 - \sqrt{3})e^{\kappa\phi}, \quad (6.19)$$

the wanted solution is

$$\kappa\langle \phi \rangle = \frac{1}{2}(a_1 + a_2) + 2, \quad \langle a \rangle = 0, \quad \kappa(a_1 - a_2) = \pm 4\sqrt{3}(2 - \sqrt{3}), \quad (6.20)$$

$$\kappa\langle z \rangle = \pm(\sqrt{3} - 1)e^{\kappa\langle \phi \rangle}, \quad (6.21)$$

and the supersymmetry is broken, most likely, softly. The unusual property for these two Polonyi vacua is that the conservation law of the scale current is now violated by the dilaton term as well as a constant term. The good news is this constant term is related to the dilaton

mass so that it still vanishes as the dilaton mass goes to zero. In this sense it still satisfies the notion of PCDC. If there is nonvanishing D-term contribution, the vacuum energy is no longer vanishing.

With one chiral multiplet we are not able to obtain an explicit vacuum solution for $V > 0$. This, however, will not be a global minimum anyway so that it is less interesting. The asymptotic vacuum at $z = 0$ and $\phi \rightarrow -\infty$, where both supersymmetry and scale symmetry are unbroken, is surrounded by a valley because $V \neq 0$ if $z \neq 0$ as $\phi \rightarrow -\infty$. All vacua with $V = 0$ are degenerate with the asymptotic vacuum so that there could be nonperturbative states associated. We expect there are a lot more interesting structures hidden in the case with more than one chiral multiplet, and we will report the results elsewhere in the near future.

7. Conclusions

I have defined the scale symmetry in the flat spacetime using the diffeomorphism structure of the supersymmetric dilaton geometry. It naturally incorporates the dilaton-axion multiplet in the superconformal generalization of supersymmetric gauge theories. Explicit examples of spontaneous breaking of the scale symmetry have been demonstrated. Depending on the details of the dilaton structure, both vacua with or without the Poincaré supersymmetry can be obtained.

I have only analyzed the cases that allow asymptotic superconformally invariant vacuum. However, other possibilities can also be speculated. For example, if there is a mechanism to lead to the dilatino condensation, then such an asymptotic vacuum can be completely destabilized so that the vacua with a vanishing vacuum energy we obtained will not run away toward the asymptotic vacuum.

One disadvantage (or perhaps, it might turn out to be a new discovery.) is, if we demand the low energy scale symmetry is the same as the scale symmetry in gravity, the Diff symmetry of curved spacetime must appear as spontaneously broken down to the volume-preserving diffeomorphism (SDiff) symmetry, which nevertheless still contains the Poincaré symmetry. Although this is nothing against any current experimental observations, some readers may find it very difficult to accept it. We hope nature herself will clarify this in future.

$N = 2$ generalization is expected to address the results obtained in Seiberg-Witten model[29] in the line of [26]. In the $N = 1$ examples given the dilaton-axion enters holomorphically and the details of the dilaton-axion produces both broken and unbroken supersymmetry. Thus we

may be able to reproduce similar results in this approach. If there is a way to incorporate the gauge coupling constant τ with ϕ_c , it is consistent with the SW model. In my approach, this is impossible if the theory is defined in four dimensions alone. However, compactifications from a higher dimensional construction would allow the identification of $\phi_c = \tau$ because the phenomenon of no direct coupling between gauge fields and the dilaton is unique to four dimensions. If this turns out to be true, then we can identify the Peccei-Quinn symmetry as the R-symmetry in the supersymmetric scale symmetry. Furthermore, we might be able to obtain QCD from the SW model, explicitly realizing the line of idea as in [26]. I hope to report the progress in the near future.

Acknowledgements: I thank R. Arnowitt, D.Z. Freedman, M. Grisaru and G. Mack for helpful correspondence.

Appendix A. Supersymmetric Volume-Preserving Diffeomorphisms

The volume-preserving diffeomorphisms are defined by diffeomorphisms that leave a volume element invariant. This can be defined for any manifolds with or without a boundary. The main advantage of decomposing diffeomorphisms into volume-preserving diffeomorphisms and the rest is because the rest are in fact conformal diffeomorphisms.

In the bosonic case the defining condition is

$$\delta\sqrt{g} = \frac{1}{2}g^{\mu\nu}\delta g_{\mu\nu} = \nabla_\mu v^\mu = 0. \quad (\text{A.1})$$

In the presence of a boundary v^μ has to be also parallel with the boundary. This is the precise condition none of conformal diffeomorphisms can satisfy which requires $\nabla_\mu v^\mu \neq 0$. Thus the dilaton geometry excludes volume-preserving diffeomorphisms to deal only with the conformal diffeomorphisms.

To define the supersymmetric dilaton geometry we need to define the supersymmetric generalization of SDiff. As ordinary SDiff transformations leave the volume density \sqrt{g} invariant, supersymmetric SDiff transformations should leave the chiral density \mathcal{E} invariant, hence ψ also invariant. In curved spacetime, therefore, we also demand $\delta(\sigma_{\alpha\dot{\alpha}}^\mu\psi_\mu^\alpha) = 0$. Using $\sigma_{\alpha\dot{\alpha}}^\mu\psi_\mu^\alpha = E_a{}^\mu\sigma_{\alpha\dot{\alpha}}^a\psi_\mu^\alpha$, we obtain

$$\delta\bar{\psi}_{\dot{\alpha}} = v^\mu\partial_\mu\bar{\psi}_{\dot{\alpha}} - \frac{2}{n}\sigma_{\alpha\dot{\alpha}}^\mu\partial_\mu\zeta^\alpha = 0. \quad (\text{A.2})$$

Thus, together with eq.(A.1), this defines the supersymmetric SDiff.

References

- [1] S. Coleman, “Dilatations,” in *Aspects of Symmetry*, (Cambridge, 1985).
- [2] J.L. Friedman, H.W. Kendall and R.E. Taylor, *Rev. Mod. Phys.* **63** (1991) 573.
- [3] J. Wess, *Nuovo Cim.* **18** (1960) 1086;
- [4] G. Mack and A. Salam, *Ann. Phys.* **53** (1969) 174;
C. Callan, S. Coleman and R. Jackiw, *Ann. Phys.* **59** (1970) 42;
D.J. Gross and J. Wess, *Phys. Rev.* **D2** (1970) 753.
- [5] C.G. Callan, *Phys. Rev.* **D2** (1970) 1541;
K. Symanzik, *Comm. Math. Phys.* **18** (1970) 227.
- [6] G. Mack, *Nucl. Phys.* **B35** (1971) 592.
- [7] S.L. Adler, J.C. Collins and A. Duncan, *Phys. Rev.* **D15** (1977) 1712;
J.C. Collins, A. Duncan and S.D. Joglekar, *Phys. Rev.* **D16** (1977) 438;
N.K. Nielsen, *Nucl. Phys.* **B120** (1977) 212.
- [8] R.D. Peccei and H.R. Quinn, *Phys. Rev. Lett.* **38** (1977) 1440; *Phys. Rev.* **D16** (1977) 1791;
S. Weinberg, *Phys. Rev. Lett.* **40** (1978) 223;
F. Wilczek, *Phys. Rev. Lett.* **40** (1978) 279;
J.-E. Kim, *Phys. Rev. Lett.* **43** (1979) 103;
M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, *Nucl. Phys.* **B166** (1980) 493;
M. Dine, W. Fischler and M. Srednicki, *Phys. Lett.* **104B** (1981) 199.
- [9] E. Cremmer and B. Julia, *Nucl. Phys.* **B159** (1979) 141.
- [10] A.A. Belavin, A.M. Polyakov and A.B. Zamolodchikov, *Nucl. Phys.* **B241** (1984) 333;
D. Friedan, Z. Qiu and S. Shenker, *Phys. Rev. Lett.* **52** (1984) 1575.
- [11] E. Witten, *Phys. Lett.* **149B** (1984) 351; *Phys. Lett* **155B** (1984) 151.
- [12] D. Friedan, *Ann. Phys.* **163** (1985) 318;
E.S. Fradkin and A.A. Tseytlin, *Phys. Lett.* **158B** (1985) 316; **160B** (1985) 69;
C.G. Callan, E.J. Martinec, M.J. Perry and D. Friedan, *Nucl. Phys.* **B262** (1985) 593.
- [13] J. Polchinski, *Nucl. Phys.* **B307** (1988) 61.
- [14] H.S. La and P. Nelson, *Phys. Rev. Lett.* **63** (1989) 24; *Nucl. Phys.* **B332** (1990) 83-130.
- [15] M. Dine and N. Seiberg, *Phys. Rev. Lett.* **57** (1986) 2625.

- [16] V. Kaplunovsky and J. Louis, Phys. Lett. **B306** (1993) 269.
- [17] A. Niemeyer and H.P. Nilles, “Gaugino Condensation and the Vacuum Expectation Value of the Dilaton,” (hep-th/9508173).
- [18] H.S. La, “Dilatations Revisited,” hep-th/9608148.
- [19] W. Buchmüller and N. Dragon, “Dilatons in Flat and Curved Space-Time,” Nucl. Phys. **B321** (1989) 207.
- [20] H.S. La, “Area Preserving Diffeomorphisms and 2-d Gravity,” MIT-CTP-2462 (hep-th/9510147).
- [21] G. Mack, in *Nonperturbative Quantum Field Theory*, ed. by G. 't Hooft (Plenum, New York, 1988).
- [22] J. Wess and J. Bagger, *Supersymmetry and Supergravity*, 2nd ed. (Princeton, 1992).
- [23] J. Wess and B. Zumino, Nucl. Phys. **B70** (1974) 39.
- [24] For reviews see P. Nath, R. Arnowitt and A.H. Chamseddine, “*Applied N=1 Supergravity*,” (World Sci., Singapore, 1984); H.P. Nilles, Phys. Rep. **110** (1984) 1; H. Haber and G. Kane, Phys. Rep. **117** (1985) 75.
- [25] R. Barbieri, S. Ferrara and C.A. Savoy, Phys. Lett **119B** (1982) 343.
- [26] L. Alvarez-Gaume, J. Distler, C. Kounnas and M. Marino, hep-th/9604004; L. Alvarez-Gaume and M. Marino, hep-th/9606168; hep-th/9606191.
- [27] M.J. Sohnius, in *Supersymmetry and Supergravity 1983*, ed. B. Milewski (World Scientific, Singapore, 1983); Phys. Rep. **128** (1985) 39-204.
- [28] J. Polonyi, Univ. of Budapest Rep. KFKI-1977-93 (1977).
- [29] N. Seiberg and E. Witten, Nucl. Phys. **B426** (1994) 19, E **B430** (1994) 485; Nucl. Phys. **B431** (1994) 484.